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Diffraction of a plane acoustic wave by a non-uniform thermoelastic cylindrical layer bounded by inviscid heat-conducting fluids☆

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ABSTRACT

The diffraction of sound by a radially laminated isotropic thermoelastic cylindrical shell is considered. The system of equations for small perturbations of a hollow thermoelastic cylinder is reduced to a system of ordinary differential equations, the boundary-value problem for which is solved by the spline-collocation method. Expressions are obtained describing the wave field outside the cylindrical layer. Results of calculations of polar radiation patterns of the amplitude of the scattered acoustic wave in the far zone are presented.

The problem of the diffraction of an acoustic wave by an isotropic non-uniform solid cylindrical body was solved in Ref. 1. The scattering of a plane monochromatic acoustic wave by a transversely isotropic non-uniform cylindrical layer was investigated in Ref. 2. A solution of the problem of the diffraction of plane acoustic waves by a non-uniform hollow cylinder was obtained in the general case of cylindrical anisotropy in Ref. 3. In all these papers thermal processes in the elastic non-uniform bodies were ignored.

1. Formulation of the problem

Consider an infinite isotropic non-uniform thermoelastic hollow circular cylinder with outer radius r_1 and inner radius r_2, having a constant temperature T_0 in the unperturbed state. There are no heat sources in the cylindrical layer. A cylindrical system of coordinates r, φ, z is chosen in such a way that the z coordinate axis is the axis of rotation of the cylinder. The moduli of elasticity, the temperature coefficient of linear expansion and the thermal conductivity of the material of the layer are described by differentiable functions of the r coordinate. The density of the layer material and its volume heat capacity are described by continuous functions of the r coordinate. We will assume that the fluid surrounding the cylindrical shell and the fluid in the cavity are inviscid, heat conducting and uniform and have a temperature T_0, densities ρ_1 and ρ_2 and velocities of sound c_1 and c_2 respectively.

Suppose a plane acoustic wave, the velocity potential of which is equal to

$$\Psi_i = A_i \exp[i(\mathbf{k}_{11}\mathbf{r} - \omega t)]$$

is incident obliquely on the thermoelastic cylinder, where A_i is the amplitude of the incident wave, $\mathbf{k}_{11}$ is the wave vector, $\mathbf{r}$ is the radius vector and ω is the angular frequency. The time factor $\exp(-i\omega t)$ will henceforth be omitted. Without loss of generality we will assume that the vector $\mathbf{k}_{11}$ lies in the $\varphi = 0,\pi$ plane.

We will represent the potential of the incident wave in the cylindrical system of coordinates in the form[4]

$$\Psi_i = \exp(ik_{11}^z z) \sum_{m=0}^{\infty} \eta_m J_m(k_{11}^r r) \cos m\varphi \tag{1.1}$$

where $k_{11}^z = k_{11} \cos \theta_0$ and $k_{11}^r = k_{11} \sin \theta_0$ are the projections of the wave vector $\mathbf{k}_{11}$ on to the z and r axes respectively, $\mathbf{k}_{11}$ is the wave number of the acoustic waves in the surrounding fluid, θ_0 is the angle between the vector $\mathbf{k}_{11}$ and the z axis, $\eta_m = A_i(2 - \delta_{0m})i^m$, δ_{0m} is the Kronecker delta, and $J_m(x)$ is the cylindrical Bessel function of order m

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We will determine the waves reflected from the cylinder and excited in the cavity and we will also obtain the displacement field and the temperature in the cylindrical thermoelastic layer.

2. The wave-field equations

Small perturbations of the isotropic thermoelastic cylindrical layer are described by the general equations of motion of a continuous medium[5] in a cylindrical system of coordinates

$$\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\varphi}}{\partial\varphi}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{1}{r}(\sigma_{rr}-\sigma_{\varphi\varphi}) = \rho\ddot{u}_r$$
$$\frac{\partial\sigma_{r\varphi}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi}+\frac{\partial\sigma_{\varphi z}}{\partial z}+\frac{2}{r}\sigma_{r\varphi} = \rho\ddot{u}_\varphi$$
$$\frac{\partial\sigma_{rz}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\varphi z}}{\partial\varphi}+\frac{\partial\sigma_{zz}}{\partial z}+\frac{1}{r}\sigma_{rz} = \rho\ddot{u}_z \tag{2.1}$$

and by the heat flux equation[6]

$$\lambda_T\frac{\partial^2T}{\partial r^2}+\left(\lambda'_T+\frac{\lambda_T}{r}\right)\frac{\partial T}{\partial r}+\frac{\lambda_T}{r^2}\frac{\partial^2T}{\partial\varphi^2}+\lambda_T\frac{\partial^2T}{\partial z^2}-\gamma\,\mathrm{div}\,\dot{\mathbf{u}} = c_v\dot{T} \tag{2.2}$$

where $\sigma_{\tau k}$ are the components of the stress tensor in cylindrical coordinates, which are related to the components of the strain tensor $\varepsilon_{\tau k}$ and the change in temperature T of the unperturbed layer by the Duhamel–Neumann relations[6]

$$\sigma_{rr} = 2\mu\varepsilon_{rr}+\lambda\,\mathrm{div}\,\mathbf{u}-\beta T,\quad \sigma_{r\varphi} = 2\mu\varepsilon_{r\varphi}$$
$$\sigma_{\varphi\varphi} = 2\mu\varepsilon_{\varphi\varphi}+\lambda\,\mathrm{div}\,\mathbf{u}-\beta T,\quad \sigma_{rz} = 2\mu\varepsilon_{rz}$$
$$\sigma_{zz} = 2\mu\varepsilon_{zz}+\lambda\,\mathrm{div}\,\mathbf{u}-\beta T,\quad \sigma_{\varphi z} = 2\mu\varepsilon_{\varphi z}$$
$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r},\quad \varepsilon_{\varphi\varphi} = \frac{1}{r}\left(\frac{\partial u_\varphi}{\partial\varphi}+u_r\right),\quad \varepsilon_{zz} = \frac{\partial u_z}{\partial z}$$
$$\varepsilon_{r\varphi} = \frac{1}{2}\left[\frac{1}{r}\left(\frac{\partial u_r}{\partial\varphi}-u_\varphi\right)+\frac{\partial u_\varphi}{\partial r}\right],\quad \varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right),\quad \varepsilon_{\varphi z} = \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial\varphi}\right)$$
$$\mathrm{div}\,\mathbf{u} = \varepsilon_{rr}+\varepsilon_{\varphi\varphi}+\varepsilon_{zz} \tag{2.3}$$

Here u_r, u_φ, u_z are the components of the displacement vector $\mathbf{u}$ along the coordinate axes, $\rho=\rho(r)$ is the density of the layer material, $\lambda=\lambda(r)$ and $\mu=\mu(r)$ are the moduli of elasticity of the layer material, $\beta=\beta(r)=3\alpha_TK$, $\alpha_T=\alpha_T(r)$ is the coefficient of linear expansion of the layer material, $K=\lambda+(2/3)\mu$ is the isothermal modulus of volume expansion, $\lambda_T=\lambda_T(r)$ and $c_v=c_v(r)$ are the thermal conductivity and volume heat capacity of the layer material respectively, and $\gamma=T_0\beta$. A prime denotes a derivative with respect to r.

Since the cylinder material is only non-uniform in the radial direction, the dependence of the components of the displacement vector and of the change in temperature on the z coordinate, according to Snell's law,[7] has the form $\exp(ik_{11}^zz)$. Hence the functions u_r, u_φ, u_z, T will be sought in the form

$$u_r(r,\varphi,z) = U_1(r,\varphi)\exp(ik_{11}^zz),\quad u_\varphi(r,\varphi,z) = U_2(r,\varphi)\exp(ik_{11}^zz)$$
$$u_z(r,\varphi,z) = U_3(r,\varphi)\exp(ik_{11}^zz),\quad T(r,\varphi,z) = U_4(r,\varphi)\exp(ik_{11}^zz) \tag{2.4}$$

In the case considered u_r, u_z and T are even functions of the coordinate φ, and u_φ is an odd function of φ. The functions $U_\alpha(\alpha=1,2,3,4)$ can be represented by the following Fourier series

$$U_\alpha(r,\varphi) = \sum_{m=0}^{\infty}U_{\alpha m}(r)\cos m\varphi,\quad \alpha = 1,3,4;\quad U_2(r,\varphi) = \sum_{m=0}^{\infty}U_{2m}(r)\sin m\varphi \tag{2.5}$$

We will introduce the following dimensionless quantities

$$r^* = \frac{r}{H},\quad U_{\alpha m}^* = \frac{U_{\alpha m}}{H},\quad \alpha = 1,2,3,\quad U_{4m}^* = \frac{U_{4m}}{T_0},\quad \lambda^* = \frac{\lambda}{\lambda_0}$$
$$\mu^* = \frac{\mu}{\mu_0},\quad \rho^* = \frac{\rho}{\rho_0},\quad \alpha_T^* = \frac{\alpha_T}{\alpha_T^0},\quad \lambda_T^* = \frac{\lambda_T}{\lambda_T^0},\quad c_v^* = \frac{c_v}{c_v^0}$$

Here $H=r_1-r_2$ is the thickness of the cylindrical layer and λ_0, μ_0, ρ_0, α_T^0, λ_T^0, c_v^0 are characteristic quantities.

Substituting expressions (2.3)–(2.5) into Eqs (2.1) and (2.2) and using the conditions for the functions $\cos m\varphi$ and $\sin m\varphi$ to be orthogonal, we obtain a system of second-order linear ordinary differential equations in the functions $U^*_{\alpha m}(\alpha = 1, 2, 3, 4)$ for each value of $m = 0, 1, 2, \ldots$.

$$A\mathbf{U}'' + B\mathbf{U}' + C\mathbf{U} = 0 \tag{2.6}$$

where

$$\mathbf{U} = (U^*_{1m}, U^*_{2m}, U^*_{3m}, U^*_{4m})^T$$

$$A = \operatorname{diag}\{a_{11}, a_{22}, a_{33}, a_{44}\}, \quad B = \|b_{\alpha\beta}\|, \quad C = \|c_{\alpha\beta}\|; \quad \alpha, \beta = 1, 2, 3, 4$$

$$a_{11} = l\lambda^* + 2\mu^*, \quad a_{22} = a_{33} = \mu^*, \quad a_{44} = \lambda_T^*$$

$$b_{11} = l\lambda^{*\prime} + 2\mu^{*\prime} + \frac{l\lambda^* + 2\mu^*}{r^*}, \quad b_{12} = -b_{21} = m\frac{l\lambda^* + \mu^*}{r^*}, \quad b_{13} = b_{31} = s_1(l\lambda^* + \mu^*)$$

$$b_{14} = -l_1\beta^*$$

$$b_{22} = b_{33} = \mu^{*\prime} + \frac{\mu^*}{r^*}, \; b_{23} = b_{24} = b_{32} = b_{34} = b_{42} = b_{43} = 0, \; b_{41} = s\beta^*, \; b_{44} = \lambda_T^{*\prime} + \frac{\lambda_T^*}{r^*}$$

$$c_{11} = \frac{1}{r^*}\left(l\lambda^* - \frac{l\lambda^* + 2\mu^*}{r^*} - m^2\frac{\mu^*}{r^*}\right) + s_1^2\mu^* + q_0\rho^*$$

$$c_{12} = m\frac{1}{r^*}\left(l\lambda^{*\prime} - \frac{l\lambda^* + 3\mu^*}{r^*}\right), \quad c_{13} = s_1 l\lambda^{*\prime}, \quad c_{14} = -l_1\beta^{*\prime}$$

$$c_{21} = -m\frac{1}{r^*}\left(\mu^* + \frac{l\lambda^{*\prime} + 3\mu^*}{r^*}\right), \quad c_{22} = -\frac{1}{r^*}\left(\mu^{*\prime} + \frac{\mu^*}{r^*} + m^2\frac{l\lambda^* + 2\mu^*}{r^*}\right) + s_1^2\mu^* + q_0\rho^*$$

$$c_{23} = -c_{32} = -ms_1\frac{l\lambda^* + \mu^*}{r^*}, \quad c_{24} = m\frac{l_1\beta^*}{r^*}$$

$$c_{31} = s_1\left(\mu^* + \frac{l\lambda^* + \mu^*}{r^*}\right), \quad c_{33} = -m^2\frac{\mu^*}{r^{*2}} + s_1^2(l\lambda^* + 2\mu^*) + q_0\rho^*, \quad c_{34} = -s_1 l_1\beta^*$$

$$c_{41} = s\frac{\beta^*}{r^*}, \quad c_{42} = ms\frac{\beta^*}{r^*}, \quad c_{43} = ss_1\beta^*, \quad c_{44} = -m^2\frac{\lambda_T^*}{r^{*2}} + s_1^2\lambda_T^* + q_1 c_\upsilon^*$$

$$\beta^* = 3\alpha_T^*\left(l\lambda^* + \frac{2}{3}\mu^*\right), \quad l = \frac{\lambda_0}{\mu_0}, \quad l_1 = \alpha_T^0 T_0$$

$$s = i\frac{\omega H^2\alpha_T^0\mu_0}{\lambda_T^0}, \quad s_1 = ik_{11}^z H, \quad q_0 = \frac{\rho_0 H^2\omega^2}{\mu_0}, \quad q_1 = i\frac{\omega H^2 c_\upsilon^0}{\lambda_T^0}$$

The prime denotes a derivative with respect to r^*.

We will represent the velocity of the fluid particles on the outside ($j = 1$) and in the cavity ($j = 2$) of the cylindrical shell in the form

$$\boldsymbol{\upsilon}_j = \operatorname{grad}(\Psi_j + \Phi_j), \quad j = 1, 2$$

The acoustic wave velocity potential Ψ_j and the thermal wave velocity potential Φ_j are the solutions of the following equations

$$\Delta\Psi_j + k_{j1}^2\Psi_j = 0, \quad \Delta\Phi_j + k_{j2}^2\Phi_j = 0; \quad j = 1, 2$$

where $\Psi_1 = \Psi_i + \Psi_s$, Ψ_s is the velocity potential of the reflected acoustic wave, and k_{j1} and k_{j2} are the wave numbers at the acoustic and thermal waves respectively. In this case

$$k_{jl}^2 = \frac{-M_j - (-1)^l\sqrt{M_j^2 + 4L_jN_j}}{2N_j}, \quad j, l = 1, 2$$

where

$$L_j = \frac{\omega^2}{c_j^2}\gamma_j, \quad M_j = \left(1 - \frac{i\omega\chi_j}{c_j^2}\right)\gamma_j, \quad N_j = \frac{i\chi_j}{\omega}$$

γ_j is the ratio of the specific heat capacities of the fluid at constant pressure and volume, and χ_j is the thermal diffusivity of the fluid.

The reflected waves must satisfy the radiation conditions at infinity, while the waves in the cavity must satisfy the boundedness condition. Hence, the functions Ψ_s, Ψ_2, Φ_1, Φ_2 will be sought in the form

$$\Psi_s = \exp(ik_{11}^z z)\sum_{m=0}^{\infty} A_{1m}H_m(k_{11}^r r)\cos m\varphi, \quad \Psi_2 = \exp(ik_{21}^z z)\sum_{m=0}^{\infty} A_{2m}J_m(k_{21}^r r)\cos m\varphi \tag{2.7}$$

$$\Phi_j = \exp(ik_{j2}^z z)\sum_{m=0}^{\infty} B_{jm}Z_{jm}(k_{j2}^r r)\cos m\varphi, \ j = 1, 2; \ Z_{1m}(x) = H_m(x), \ Z_{2m}(x) = J_m(x) \tag{2.7}$$

where k_{jl}^z and $k_{jl}^r (j, l = 1, 2)$ are the projections of the wave vector $\mathbf{k}_{jl}$ onto the z and r axes respectively, $(k_{jl}^z)^2 + (k_{jl}^r)^2 = k_{jl}^z$ and $H_m(x)$ is the cylindrical Hankel function of the first kind of order m. By Snell's law $k_{11}^z = k_{12}^z = k_{21}^z = k_{22}^z$.

The coefficients A_{jm}, B_{jm} (j = 1,2) are to be determined from the boundary conditions, which consist of the equality of the normal velocities of the particles of the thermoelastic medium and of the fluid on the outer and inner surfaces of the cylindrical layer, the absence of shear stresses on these surfaces, the equality on them of the normal stress and the acoustic pressure, and the continuity of the acoustic temperature and the heat flux on the layer surfaces:

$$r = r_j: -i\omega u_r = \upsilon_{jr}, \quad \sigma_{rz} = 0, \quad \sigma_{r\varphi} = 0, \quad \sigma_{rr} = -p_j$$
$$T = T_j, \quad \lambda_T\frac{\partial T}{\partial r} = \lambda_j\frac{\partial T_j}{\partial r}; \quad j = 1, 2 \tag{2.8}$$

Here

$$\upsilon_{jr} = \frac{\partial}{\partial r}(\Psi_j + \Phi_j), \quad p_j = i\omega\rho_j(\Psi_j + \Phi_j)$$

$$T_j = \frac{1}{\alpha_j}\left[\frac{i\omega\gamma_j}{c_j^2}(\Psi_j + \Phi_j) + \frac{i}{\omega}\Delta(\Psi_j + \Phi_j)\right]; \quad j = 1, 2$$

where υ_{jr} are the normal components of the velocities of the fluid particles, p_j are the acoustic pressures, T_j are the acoustic temperatures, and α_j and λ_j are the coefficient of thermal expansion and the conductivities on the outside (j = 1) and in the cavity (j = 2) of the cylindrical shell, respectively.

Substituting expressions (1.1), (2.3), (2.4), (2.5) and (27) into boundary conditions (2.8) and using the conditions for the functions $\cos m\varphi$ and $\sin m\varphi$ to be orthogonal, for each value of index m = 0, 1, 2, . . . we obtain a system of twelve equations, from which we find expressions for the coefficients A_{jm} and B_{jm} (j = 1, 2):

$$\mathbf{X}_j = E_j\mathbf{Y}|_{r^* = r_j^*}, \quad j = 1, 2 \tag{2.9}$$

and eight conditions for finding a particular solution of system of differential equations (2.6)

$$(A\mathbf{U}' + G_j\mathbf{U})|_{r^* = r_j^*} = \mathbf{D}_j, \ j = 1, 2 \tag{2.10}$$

where

$$\mathbf{X}_j = (A_{jm}, B_{jm})^T, \quad \mathbf{Y} = (U_{1m}^*, U_{4m}^*, \eta_m)^T, \quad \mathbf{D}_j = (d_1\delta_{1j}, 0, 0, d_4\delta_{1j})^T$$
$$E_j = \|e_{li}^j\|; \quad l = 1, 2, \quad i = 1, 2, 3, \quad G_j = \|g_{\alpha\beta}^j\|; \quad \alpha, \beta = 1, 2, 3, 4$$

Here

$$d_1 = -\frac{i\omega\rho_1}{\mu_0}[J_m(x_{11}) + e_{13}^1 H_m(x_{11}) + e_{23}^1 H_m(x_{12})]\eta_m$$

$$d_4 = -\frac{z_1}{r^*}[\xi_{11}x_{11}J'_m(x_{11}) + e_{13}^1\xi_{11}x_{11}H'_m(x_{11}) + e_{23}^1\xi_{12}x_{12}H'_m(x_{12})]\eta_m$$

$$e_{11}^j = \frac{i\omega H^2 r^* \xi_{j2} Z_{jm}(x_{j2})}{w_j}, \quad e_{12}^j = -\frac{i\alpha_j T_0 x_{j2} Z'_{jm}(x_{j2})}{w_j}$$

$$e_{13}^j = -\left(\frac{J'_m(x_{11})}{H'_m(x_{11})} + \frac{2i\xi_{11}}{\pi w_1}\frac{x_{12}H'_m(x_{12})}{x_{11}H'_m(x_{11})}\right)\delta_{1j}$$

$$e_{21}^j = -\frac{i\omega H^2 r^* \xi_{j1} Z_{jm}(x_{j1})}{w_j}, \quad e_{22}^j = \frac{i\alpha_j T_0 x_{j1} Z'_{jm}(x_{j1})}{w_j}, \quad e_{23}^j = \frac{2i\xi_{11}}{\pi w_1}\delta_{1j}$$

$$r_j^* = \frac{r_j}{H}, \quad x_{jl} = k_{jl}^r r_j, \quad z_j = -\frac{i\lambda_j}{\alpha_j T_0 \lambda_T^0}, \quad \xi_{jl} = \frac{\omega\gamma_j}{c_j^2} - \frac{k_{jl}^2}{\omega}, \quad l = 1, 2$$

$$w_j = \xi_{j1} x_{j2} Z_{jm}(x_{j1}) Z'_{jm}(x_{j2}) - \xi_{j2} x_{j1} Z'_{jm}(x_{j1}) Z_{jm}(x_{j2})$$

$$g_{11}^j = \frac{l\lambda^*}{r^*} + \frac{i\omega\rho_j}{\mu_0}[e_{11}^j Z_{jm}(x_{j1}) + e_{21}^j Z_{jm}(x_{j2})], \quad g_{12}^j = m\frac{l\lambda^*}{r^*}$$

$$g_{13}^j = s_1 l\lambda^*, \quad g_{14}^j = -l_1\beta^* + \frac{i\omega\rho_j}{\mu_0}[e_{12}^j Z_{jm}(x_{j1}) + e_{22}^j Z_{jm}(x_{j2})]$$

$$g_{21}^j = -m\frac{\mu^*}{r^*}, \quad g_{22}^j = -\frac{\mu^*}{r^*}, \quad g_{23}^j = g_{24}^j = g_{32}^j = g_{33}^j = g_{34}^j = g_{42}^j = g_{43}^j = 0,$$

$$g_{31}^j = s_1\mu^*$$

$$g_{41}^j = \frac{z_j}{r^*}[e_{11}^j \xi_{j1} x_{j1} Z'_{jm}(x_{j1}) + e_{21}^j \xi_{j2} x_{j2} Z'_{jm}(x_{j2})]$$

$$g_{44}^j = \frac{z_j}{r^*}[e_{12}^j \xi_{j1} x_{j1} Z'_{jm}(x_{j1}) + e_{22}^j \xi_{j2} x_{j2} Z'_{jm}(x_{j2})]$$

It follows from system (2.9) that the coefficients A_{jm} and B_{jm} can only be calculated after determining the values of the functions $U_{1m}^*(r*)$ and $U_{4m}^*(r*)$ on the surfaces of the cylindrical shell.

3. Solution of the boundary-value problem by the spline-collocation method

To find the functions $U_{1m}^*(r*)$, $U_{4m}^*(r*)$ we need to solve boundary-value problem (2.6), (2.10). We will obtain the solution of this problem by the spline-collocation method.[8] In the section r_2^*, r_1^* we introduce a uniform grid $r_2^* = x_0 < x_1 < \ldots x_N = r_1^*$ with pitch h. We will seek an approximate solution of the boundary-value problem in the form of cubic splines $S_{\alpha m}(r^*)$ (α = 1,2,3,4) of defect 1 with nodes on the grid. Here $S_{\alpha m}(r^*)$ are spline-functions, which approximate the functions $U_{\alpha m}^*(r^*)$ respectively. We will represent the cubic splines in the form of an expansion in a basis of normalized cubic B-splines[8]

$$S_{\alpha m}(r^*) = \sum_{k=-1}^{N+1} b_{\alpha m}^k B_k(r^*), \quad \alpha = 1, 2, 3, 4 \tag{3.1}$$

where $b_{\alpha m}^k$ are the coefficients of the expansion, to be determined, and $V_k(r^*)$ is the basis spline-function, defined in the interval-carrier with middle node x_k. In order that all the basis functions in (3.1) should be defined, the grid must be supplemented by the nodes

$$x_{\gamma-3} = x_0 + (\gamma - 3)h, \quad x_{N+3-\gamma} = x_N + (3-\gamma)h, \quad \gamma = 0, 1, 2$$

We will require that the splines $S_{\alpha m}(r^*)$ should satisfy system (2.6) and boundary conditions (2.10) at collocation nodes, which coincide with the nodes of the grid. Using expressions for the nodal values of the B-spline and its derivatives,[8] we obtain a system of linear algebraic equations in the unknown coefficients of the expansion

$$P_{0m}\mathbf{b}_{0m} = 0, \quad Q_{km}\mathbf{b}_{km} = 0, \quad k = 0, 1, \ldots, N, \quad R_{Nm}\mathbf{b}_{Nm} = \mathbf{S}_{Nm}$$

$$\mathbf{b}_{km} = (b_{1m}^{k-1}, b_{2m}^{k-1}, b_{3m}^{k-1}, b_{4m}^{k-1}, b_{1m}^k, b_{2m}^k, b_{3m}^k, b_{4m}^k, b_{1m}^{k+1}, b_{2m}^{k+1}, b_{3m}^{k+1}, b_{4m}^{k+1})^T \tag{3.2}$$

where S_{Nm} is a vector, consisting of four components, and P_{0m}, Q_{km} and R_{Nm} are 4×12 matrices.

Solving system (3.2), consisting of $4N + 12$ equations in $4N + 12$ unknown coefficients, and substituting the values obtained into expression (3.1), we obtain an approximate solution of the boundary-value problem.

Determining the coefficients A_{jm} and B_{jm} (j = 1,2) from expressions (2.9), we obtain a description of the wave fields on the outside and in the cavity of the cylindrical shell from formulae (2.7).

4. Results of calculations

We will consider the far zone of the acoustic field. Using the asymptotic representation of the cylindrical Hankel functions for large values of the argument, we obtain the following expression for the potential of the scattered acoustic wave

$$\Psi_s \approx \sqrt{\frac{r_1}{2r}}\exp\left[i\left(k_{11}^r r - \frac{\pi}{4}\right)\right]\exp(ik_{11}^z z)F(\varphi)$$

where

$$F(\varphi) = \frac{2}{\sqrt{\pi k_{11}^r r_1}}\sum_{m=0}^{\infty}(-i)^m A_m \cos m\varphi$$

Using the solution of the problem obtained we calculated the angular dependences of the amplitude ($|F(\varphi)|$) of the scattered acoustic wave for shells situated in water with the following values of the physical parameters

$$\rho_1 = \rho_2 = 1000\ \text{kg/m}^3, \quad c_1 = c_2 = 1485\ \text{m/s}, \quad \alpha_1 = \alpha_2 = 2.1\cdot 10^{-4}\ \text{l/K}$$
$$\lambda_1 = \lambda_2 = 0.59\ \ \text{W/(m}\cdot\text{k)}$$
$$\chi_1 = \chi_2 = 1.43\cdot 10^{-7}\ \text{m}^2\text{/s}, \quad \gamma_1 = \gamma_2 = 1.006, \quad T_0 = 293\ \text{K}$$

We considered the case of the incidence of a plane acoustic wave of unit amplitude along the direction of the normal to the generatrix of the cylinder for a ratio of the outer radius of the cylindrical layer to the inner radius r_2, equal to two. The types of material of the shells were defined by the physical-mechanical characteristics shown in Table 1.

Material of type A is similar in its physical-mechanical characteristics to metals (aluminium), while material of type B is similar to polymers (polyvinyl butyral).

Calculations were carried out both for uniform materials and for materials, the dimensionless density and dimensionless modulus of elasticity of which varied simultaneously over the thickness of the cylindrical layer in accordance with the law

$$f(r^*) = a\{0.2 + \exp[-4(r^* - r_2^*)^2/(r_1^* - r_2^*)^2]\}$$

The factor a is chosen so that the mean value of the function $f(r^*)$ over the thickness of the layer was equal to unity.

To estimate the effect of the thermoelasticity of the material of the cylindrical shells on the scattering of the sound, calculations were also carried out for elastic shells in the case of an isothermal process.

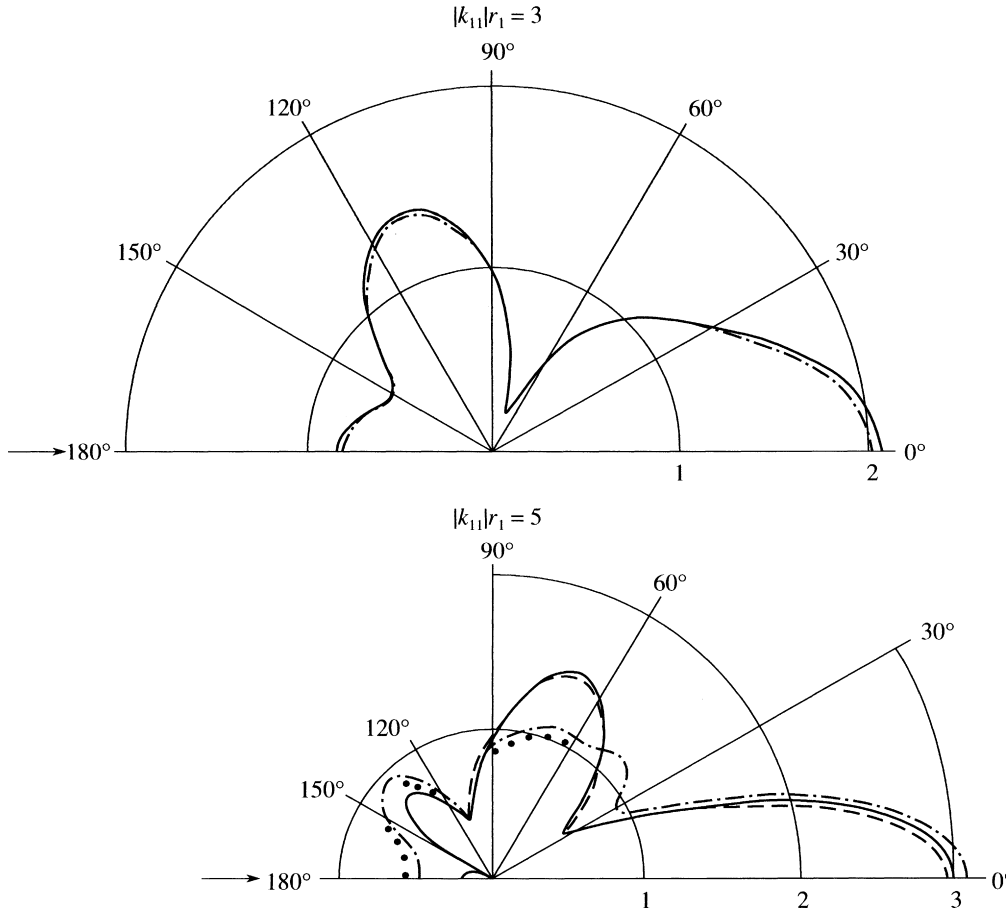
In Fig. 1 we show, for material of type A, polar directional patterns of the amplitude of the scattered acoustic wave for wave dimensions $|k_{11}|r_1 = 3$ and $|k_{11}|r_1 = 5$ of the cylindrical shell. In view of the symmetry of the scattered acoustic field the diagrams were constructed for a polar angle in the range $0^\circ \le \varphi \le 180^\circ$. Here and henceforth continuous curves correspond to a uniform elastic material, dashed curves correspond to a uniform thermoelastic material, dash-dot curves correspond to a non-uniform elastic material, and dotted curves correspond to a non-uniform thermoelastic material. The direction of propagation of the incident plane wave is shown by the arrow. It follows from the graphs that when $|k_{11}|r_1 = 3$ the thermoelasticity of the shell material has no effect on the scattering of the sound. The effect of non-uniformity of the material is weak and gives rise to only a small reduction in the value of the sidelobes of the pattern around their maxima. When the wave dimension of the shell is increased ($|k_{11}|r_1 = 5$) the effect of the thermoelasticity becomes somewhat more noticeable. The presence of non-uniformity in the cylindrical layer for the same frequency leads to deformation of the form of the directional pattern. The size of the sidelobes in the illuminated and transition zones is changed considerably and, moreover, the lobe in the direction $\varphi = 180^\circ$ disappears.

In Fig. 2 we show polar radiation patterns of the amplitude of the scattered acoustic wave for cylindrical shells made of material of type B, for the same values of $|k_{11}|r_1$ as in Fig. 1. An analysis of this patterns shows that, unlike the thermoelasticity of the material of type A, the thermoelasticity of type B material has a much greater effect on $|F(\varphi)|$. Thus, in the neighbourhood of $\varphi = 0^\circ$ for $|k_{11}|r_1 = 3$ and in the illuminated zone for $|k_{11}|r_1 = 5$ the thermoelasticity of type B material leads to a considerable increase in $|F(\varphi)|$. If the variability of the density and the modulus of elasticity of the type B material is taken into account, the shape of the polar radiation pattern changes, which is most noticeable when $|k_{11}|r_1 = 3$, when the maximum value in the lobe for $\varphi = 0^\circ$ is almost doubled and a new large lobe occurs in the region of $\varphi = 70^\circ$. When $|k_{11}|r_1 = 5$ the radiation pattern undergoes the greatest change in the illuminated\one where one lobe appears instead of two. The radiation pattern for $|k_{11}|r_1 = 5$ shows that non-uniformity of the material of the scatterer may change the effect of the thermoelasticity on the scattering amplitude of the sound. For $\varphi = 0^\circ$ in a uniform material, thermoelasticity leads to a reduction in the amplitude, while in a non-uniform material it leads to an increase.

Hence an analysis of the results of numerical calculations shows that the thermoelasticity of the material of the cylindrical layer, like its non-uniformity, has a considerable effect on the scattering of sound, and the extent of this effect depends very much on the type of material.

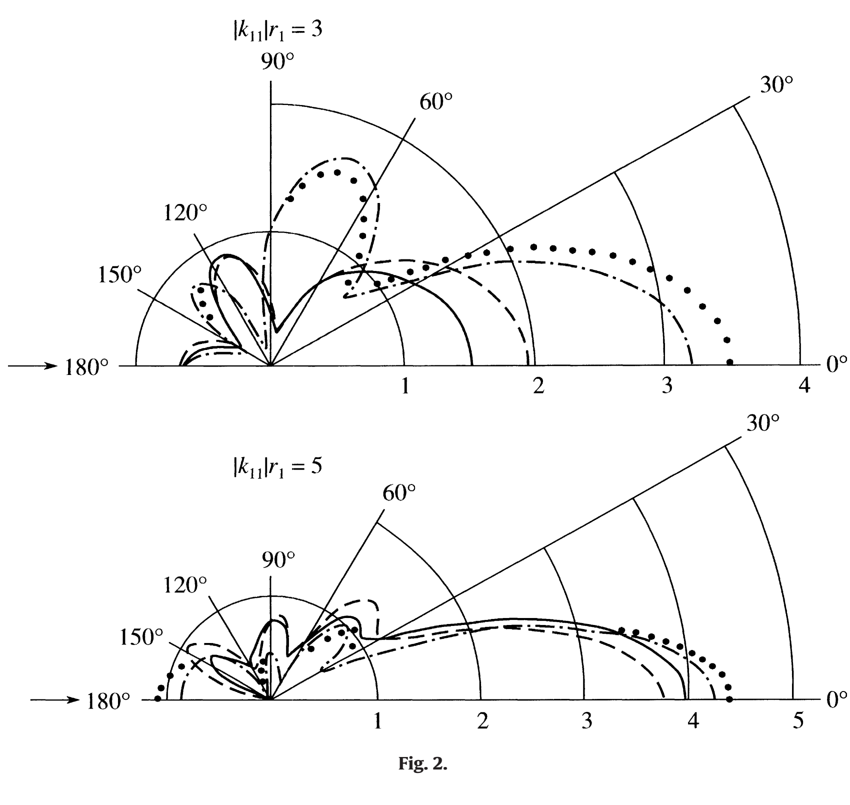
Table 1

Type of material	λ_0, N/m^2	μ_0, N/m^2	ρ_0, kg/m^3	c_v^0, J/(m^3·K)	α_T^0, 1/K	λ_T^0, W/(m·K)
A	$5.6\cdot 10^{10}$	$2.6\cdot 10^{10}$	2700	$2.3\cdot 10^{6}$	$25.5\cdot 10^{-6}$	236
B	$3.9\cdot 10^{9}$	$9.8\cdot 10^{8}$	1700	$1.2\cdot 10^{6}$	$2.3\cdot 10^{-4}$	0.2



$|k_{11}|r_1 = 3$
90°
120°
60°
150°
30°
180°
0°
1
2
$|k_{11}|r_1 = 5$
90°
60°
30°
120°
150°
180°
0°
1
2
3

Fig. 1.



$|k_{11}|r_1 = 3$
90°
60°
30°
120°
150°
180°
0°
1
2
3
4
$|k_{11}|r_1 = 5$
30°
60°
90°
120°
150°
180°
0°
1
2
3
4
5

Fig. 2.

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